Q.1 (a) Define Kernel, Rank and Nullity?          (5)

(b) There are three events A, B, C. One of which must, and only one can happen. The odds are 8 to 3 against A, 5 to 2 against B; find the odds against C.          (5)

(c) If X is a Poisson variable such that \( P(X=2) = 9P(X=4) + 90P(X=6) \), find the mean and variance of X.          (5)

(d) What is pivoting? Explain in terms of Gauss–Elimination method.          (5)

Section – A

Q.2 (a) Define vector space. Let \( F(x) \) be a set of all polynomials of an arbitrary field \( F \), then show that \( F(x) \) is a vector space with ordinary addition of polynomials and multiplication of polynomials by an element of \( F \).          (10)

(b) Define basis of a vector space. Also prove that the following vectors \( (1,0,−1), (1,2,1), (0,−3,2) \) form a basis for \( R_3(\mathbb{R}) \).          (10)
Q.3 (a) A can hit a target 4 times in 5 shots; B 3 times in 5 shots; C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit? (5)

(b) X can solve 80% of the problem given in a book and Y can solve 60%, what is the probability that

(i) None will be able to solve a problem

(ii) At least one of them will solve a problem

(c) A manufacturing firm produces pipes in two plants 1 and 2 with daily production of 1,500 and 2,000 pipes respectively. The fraction of the defective pipes produced by two plants 1 and 2 is 0.006 and 0.008 respectively. If a pipe is selected at random from the day’s production. It is found to be defective. What is the chance that it has come from plant 1 and plant 2? (10)

Q.4 (a) If 20% of the memory chips made in a certain plant are defective, what are the probabilities that in a lot of 100 randomly chosen for inspection

(i) At most 15 will be defective?

(ii) Exactly 15 will be defective?

(b) Fit a binomial distribution to the following data:- (10)

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>28</td>
<td>62</td>
<td>46</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

Q.6 (a) For following data using backward difference polynomials. Interpolate at x=0.25 (10)

<table>
<thead>
<tr>
<th>X</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(x)</td>
<td>1.4</td>
<td>1.56</td>
<td>1.76</td>
<td>2.00</td>
<td>2.28</td>
</tr>
</tbody>
</table>

(b) Fit a least square parabola of the form \( y = a + bx \) to the data given below: (10)

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>14</td>
<td>27</td>
<td>40</td>
<td>55</td>
<td>68</td>
</tr>
</tbody>
</table>

Q.7 (a) Solve \( \frac{dy}{dx} = \frac{x^2}{2y} \) given that x=0, y=1.2, find y at x=0.4, take step size h=0.4 by using Runge-Kutta method. (10)

(b) Solve the following differential equation using Modified Euler’s method for the given boundary condition

\( \frac{dy}{dx} = x^3 + y \), \( y(0) = 1 \), find value of y at x=0.1 (10)

Section – B

Q.5 Use the Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize \( z = 8x_1 + 10x_2 - x_1^2 - x_2^2 \) subject to the constraints: \( 3x_1 + 2x_2 \leq 6 \), \( x_1, x_2 \geq 0 \) (20)