B.Tech 1st Semester Examination
Subject - Applied Mathematics-I
Subject Code - AHL103

Time Allowed: 03 hours. Maximum Marks: 100

Before answering the question paper the candidate should ensure that they have been supplied the correct question paper. Complaints in this regard, if any, shall not be entertained after the examination.

Note: Attempt any five questions and all questions carry equal marks.

1 (a) Find the inverse of a matrix A, by elementary transformation,

\[
A = \begin{bmatrix}
0 & 1 & 2 \\
1 & 2 & 3 \\
3 & 3 & 1
\end{bmatrix}
\]  (6)

(b) For what values of \( \lambda \) and \( \mu \), the systems of equations

\[
x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu
\]

have

(i) No Sol. (ii) Unique Sol. (iii) Infinite no. of Sols.  (8)

(c) Find the Eigen values of the matrix

\[
A = \begin{bmatrix}
8 & -6 & 2 \\
-6 & 7 & -4 \\
2 & -4 & 3
\end{bmatrix}
\]  (6)

2 (a) If \( y = \sin(m \sin^{-1}x) \), Prove that

\[
(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0
\]  (8)

(b) Find all the asymptotes of

\[
x^3 + 3x^2y - 4y^3 - x + y + 3 = 0
\]  (6)

(c) Find the Radius of curvature of the curve \( y^2 = 4ax \)  (6)

3 (a) Solve:

\[
(xy \sin xy + \cos xy)y' = (xy \sin xy - \cos xy)xy = 0
\]  (6)

(b) The charge \( q \) on the plate of a condenser of capacity \( C \) charged through a resistance \( R \) by steady voltage \( V \) satisfies the differential equation: \[ R \frac{dq}{dt} + \frac{q}{C} = V. \]

If \( q = 0 \) at \( t = 0 \), show that \( q = CV \left(1 - e^{-\frac{t}{RC}}\right) \)  (8)

(c) Find the Orthogonal trajectories of the family of curve \( xy = C \)  (6)

4 (a) Examine the convergence of the series

\[
\Sigma (3\sqrt{n^3} + 1 - n)
\]  (10)

(b) Discuss the convergence of the series

\[
\frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \frac{1.35 x^7}{7} + \ldots \ldots \ldots
\]  (10)

5 (a) If \( u = \sin^{-1} \left(\frac{x + 2y + 3z}{\sqrt{x^2 + y^2 + z^2}}\right) \), Show that

\[
x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} + 3 \tan u = 0
\]  (6)

(b) Find the extreme value of the function

\[
x^3 + y^3 - 3axy
\]  (8)

(c) If \( u = xyz, v = x^2 + y^2 + z^2, w = x + y + z, \)

find the Jacobian \( \frac{\partial(u,v,w)}{\partial(x,y,z)} \)  (6)

6 (a) Form the Partial differential equation of the function

\[
xyz = \phi(x + y + z).
\]  (6)

(b) Solve:

\[
x^2 (y - z)p + y^2 (z - x)q = z^2 (x - y), \text{ where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}
\]  (8)

(c) Solve: \[ p^2 - q^2 = x - y \]  (6)