1 (a) Prove that \[ \int_0^\frac{\pi}{4} \sqrt{\tan \theta} \, d\theta = \frac{1}{2} \Gamma \left( \frac{3}{4} \right) \Gamma \left( \frac{1}{4} \right) \] (5)

(b) Find the third difference with the arguments 2, 4, 9, 10 of the function \( f(x) = x^3 - 2x \) using Newton’s Divided Differences. (5)

(c) Find a root of the equation \( f(x) = 8x^3 - 2x - 1 = 0 \), using the Bisection Method in three iterations. (10)

SECTION - A

2 (a) Evaluate \[ \iiint \ x y (x^2 + y^2)^{3/2} \, dx \, dy \] over the positive quadrant of the circle \( x^2 + y^2 = 1 \). (10)

(b) Evaluate \[ \int_0^1 \int_0^1 e^{x+y+z} \, dx \, dy \] . (10)

3 (a) In what direction from (3, 1, -2) is the directional derivative of \( f = x^2 y z^4 \) is maximum and what is the maximum value. (10)

(b) If \( \vec{F} = (x + y + 1) \hat{i} + (x + y) \hat{j} - (x + y) \hat{k} \), show that \( \vec{F}. \text{Curl} \vec{F} = 0 \) (10)

4 (a) By the method of the least squares, find the curve \( y = ax + bx^2 \) that best fits the following data: (10)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.8</td>
<td>5.1</td>
<td>8.9</td>
<td>14.1</td>
<td>19.8</td>
</tr>
</tbody>
</table>

(b) The value of \( x \) and \( y \) are given as below: (10)

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

Find the value of \( y \) at \( x = 10 \) using Lagrange’s formula.

SECTION – B

5(a) Find the real root of the equation \( x^4 - x - 10 = 0 \), by Newton – Raphson’s Method up to third Approximation only. (10)

(b) Find a real root of the equation \( x^6 - x^4 - x^3 - 1 = 0 \) by Secant Method up to third approximation only. (10)

6. Solve the following equation by the method of factorization: (20)

\[ 3x + 2y + 7z = 4 \]
\[ 2x + 3y + z = 5 \]
\[ 3x + 4y + z = 7 \]

7 (a) Following runs are scored by two batsmen A and B in a series of innings: (10)

A: 5 26 97 76 112 89 6 108 24 16
B: 51 47 36 60 58 39 44 42 71 50

Find their mean scores and standard deviation. Who is better run scorer and who is more consistent?

(b) Calculate the mean and variance for the following data: (10)

<table>
<thead>
<tr>
<th>Class:</th>
<th>30 – 40</th>
<th>40 – 50</th>
<th>50 – 60</th>
<th>60 – 70</th>
<th>70 – 80</th>
<th>80 – 90</th>
<th>90 – 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency:</td>
<td>3</td>
<td>7</td>
<td>12</td>
<td>15</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>