Diploma / Integrated B.Tech 1st Semester Examination
June - July 2013
MATHEMATICS — I

Time Allowed: 03 hours. Maximum Marks: 100

Before answering the question paper the candidate should ensure that they have been supplied the correct question paper. Complaints in this regard, if any, shall not be entertained after the examination.

Note: Attempt any five questions and all questions carry equal marks.

Section – A

1. (a) Solve the equations using Cramer’s rule :

\[ 3x + y + 2z = 3, \quad 2x - 3y - z = -3, \quad x + 2y + z = 4 \]

(b) Prove without expanding that determinant \( \Delta = 0. \)

(c) Find the middle term or terms in expansion of \( (x + \frac{1}{x})^{12}. \)

2. (a) Find the ad joint of the following matrix:

\[
\begin{bmatrix}
1 & 0 & -1 \\
3 & 4 & 5 \\
0 & -6 & -7
\end{bmatrix}
\]

(b) Verify \( AB = BA \) and each equal to \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \) if:

\[
A = \begin{bmatrix}
-2 & 3 & -1 \\
-1 & 2 & -1 \\
-6 & 9 & -4
\end{bmatrix}
\quad \text{and} \quad B = \begin{bmatrix}
1 & 3 & -1 \\
2 & 2 & -1 \\
3 & 0 & -1
\end{bmatrix}
\]

3. (a) Prove that \( \sin 18^\circ = \frac{\sqrt{5} - 1}{4} \). Also show that \( \cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}. \)

(b) If \( A + B + C = 180^\circ \), prove the following identities:

\[ \sin 2A - \sin 2B + \sin 2C = 4 \cos A \cos C \sin B. \]

Section – B

4(a) Prove that the following points are collinear \((2,3,0), (-1,3,0), (1,3,0).\)

(c) If \( \vec{a}, \vec{b}, \vec{c} \) are non-coplanar vectors. Show that the points

\[ (6\vec{a} + 2\vec{b} - \vec{c}), (2\vec{a} - \vec{b} + 3\vec{c}), (-\vec{a} + 2\vec{b} - 4\vec{c}), (-12\vec{a} - \vec{b} - 3\vec{c}) \]

are coplanar.

5. (a) Find the modulus and argument of the following complex number \( \left( \frac{-\sqrt{3}}{2} + i \frac{1}{2} \right). \)

(b) If \( (x^2y - 2) + i(x + 2xy - 5) = 0. \) Find ‘x’ and ‘y’.

(c) Find the square root of \(-5 + 12i\).

6. (a) Four cards are drawn from a pack of cards. Find the probability that:

(i) All are diamonds.
(ii) There is one card of each suit.
(iii) There are two spades and two hearts.

(b) A bag ‘A’ contains 2 black and 3 blue balls and a bag ‘B’ contains 4 black and 5 blue balls. One ball is drawn at random from one of the bags and is found to be blue. Find the probability that it was drawn from bag ‘B’.