BCA 2nd Semester Examination
June-2014
Mathematics - II
Subject Code: AHL110
Time Allowed: 03 hours. Maximum Marks: 100

Before answering the question paper the candidate should ensure that they have been supplied the correct question paper. Complaints in this regard, if any, shall not be entertained after the examination.

Note: Attempt five questions in total. Question No.1 is compulsory and remaining four questions from Section A and B (Two questions from each section)

Q.1 (a) Evaluate \[ \lim_{x \to 2} \left( \frac{1}{x-2} - \frac{2}{x^2-4} \right) \] \hspace{1cm} (5)

(b) If \( x = \sin \theta, y = \sin n\theta \) then prove that
\[ (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + n^2y = 0. \] \hspace{1cm} (5)

(c) Evaluate \[ \int_1^x \frac{3 \cos (\log x)}{x} \, dx \] \hspace{1cm} (5)

(d) Find the centroid of the triangle whose angular points are (3,3), (-7,4) and (10,2). \hspace{1cm} (5)

Section –A

Q.2 (a) If \( y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \text{to } \infty}}} \)
Then prove that \( (2y-1) \frac{dy}{dx} = \frac{1}{x} \). \hspace{1cm} (10)

(b) Differentiate \[ \frac{x^3}{1+x^3} \] with respect to \( x^3 \). \hspace{1cm} (10)

Q.3 (a) Solve the differential equation
\[ \sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0 \] \hspace{1cm} (10)

(b) Find the maximum and minimum values of the function \( 2x^3 - 15x^2 + 36x + 10 \). \hspace{1cm} (10)

Q.4 (a) Evaluate \( \int \frac{x+2}{(x-1)(x-2)} \, dx \) \hspace{1cm} (10)

(b) Evaluate \( \int_1^a \int_1^b \frac{dydx}{xy} \) \hspace{1cm} (10)

Section –B

Q.5 (a) Solve the following linear differential equation:
\[ \frac{dy}{dx} + 2y = e^{4x}, x > 0. \] \hspace{1cm} (10)

(b) Solve the following linear differential equation:
\[ \frac{dy}{dx} + \frac{2}{x} \cdot y = x \log x \] \hspace{1cm} (10)

Q.6 (a) Find the equation of a straight line passing through the origin and passing through the point (3,-4). \hspace{1cm} (5)

(b) Find the perpendicular distance of the point (3, 4) from the line \( 12x - 5y + 7 = 0 \). \hspace{1cm} (5)

(c) Find ‘a’ and ‘b’ so that the lines \( ax+8y+b=0 \), and \( 2x+ay-1=0 \) may be,
(i) Co-incident (ii) perpendicular. \hspace{1cm} (10)

Q.7 (a) Write down the equation of ellipse whose semi major axis is 8 and the eccentricity is 0.5 \hspace{1cm} (5)

(b) Find the equation of the circle which passes through the points (2, 3) and (6, -1) and whose radius is 4. \hspace{1cm} (5)

(c) Find the eccentricity, coordinates of foci, equations of directrices and the length of latus-rectum of the ellipse \( y^2 = 4x \). \hspace{1cm} (10)