Integrated B.Tech & Diploma 1st Semester Examination
Subject - Mathematics-I
Subject Code - AHL003

Before answering the question paper the candidate should ensure that they have been supplied the correct question paper. Complaints in this regard, if any, shall not be entertained after the examination.

Note: Attempt any five questions and all questions carry equal marks.

1 (a) Find the twenty fifth term of the A.P: 12, 16, 20, 24, ……
(b) Using binomial theorem, compute \( (99)^5 \)
(c) Solve the systems of equations, by Cramer’s rule:
\[
\begin{align*}
   x + y + z &= 3, \\
   x + 2y + 3z &= 4, \\
   x + 4y + 9z &= 6
\end{align*}
\]

2 (a) Define the matrix. Find A+B, AB, where
\[
A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}, \\
B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix}
\]
(b) Find the inverse of a matrix A, where
\[
A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}
\]

3 (a) Find the value of \( \cos 100^\circ + \cos 20^\circ \)
(b) Find \( \sin \theta \) and \( \tan \theta \), if \( \cos \theta = \frac{12}{13} \)
(c) Prove that: \( 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0 \)

4 (a) Find \( \vec{a} \vec{b} \) and \( \vec{a} \times \vec{b} \),
\[
\text{when } \vec{a} = 2\hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}
\]
(b) Find the angle between the vectors
\[
5\hat{i} + 3\hat{j} + 4\hat{k} \text{ and } 6\hat{i} - 8\hat{j} - \hat{k}.
\]
(c) Find \( (\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b}) \),
\[
\text{if } \vec{a} = \hat{i} + \hat{j} + 2\hat{k} \text{ and } \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}.
\]
(d) Given \( |\vec{a}| = 10, |\vec{b}| = 2 \) and \( \vec{a} \vec{b} = 12 \), find \( |\vec{a} \times \vec{b}| \)

5 (a) Evaluate \[
\left( \frac{1}{1-2i} + \frac{3}{1+i} \right) (\frac{3+4i}{2-4i})
\]
(b) Evaluate \[
\frac{1}{1- \cos \theta + 2i \sin \theta}
\]
(c) Find the value of \( x \) and \( y \), if \( \frac{x-1}{3+i} + \frac{y-1}{3-i} = i \)

6 (a) What is the probability of 53 Sundays in a leap year
(b) Using Binomial distribution, if 20% of the bolts produced by a machine are defective. Determine the probability that out of 4 bolts chosen at random atmost 2 bolts will be defective.
(c) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total output. Out of these, 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?